

## Topic 4 Point and interval estimate

---

### Population parameter – parameter základného súboru

#### *Definition:*

A population characteristics, such as population mean, population variance and population standard deviation.

---

### Sample statistics – výberová štatistika

#### *Definition:*

A sample characteristics, such as sample mean, sample variance and sample standard deviation. **The value of a sample statistics is used to estimate the value of the population parameter.**

---

### Central limit theorem – centrálna limitná teoréma

#### *Definition:*

A theorem that allows us to use the normal probability distribution to approximate the sampling distribution of a sample mean and sample standard deviation whenever the sample size is large.

---

### Point estimate – bodový odhad

#### *Definition:*

Point estimate is a single numerical value used as an estimate of a population parameter.

#### *Note:*

*Point estimator – sample statistics, such as  $\bar{x}$ ,  $s_1$ ,  $s_1^2$  that provides a point estimate of the population parameter. Point estimate is more precise than interval estimate but it is not so reliable.*

---

### Properties of point estimators – vlastnosti bodového odhadu

#### *Definition:*

Properties of point estimators are as follows:

- **unbiasedness** – a property of point estimator that occurs whenever the expected value of the point estimator is equal to the population parameter it estimates.
  - **consistency** – a property of point estimator that occurs whenever larger sample sizes tend to provide point estimates closer to the population parameter.
  - **(relative) efficiency** – if we have two unbiased point estimators of the same population parameter, the point estimator with the smaller variance is said to have greater efficiency than the other.
- 

### Point estimate of mean – bodový odhad priemeru (strednej hodnoty)

$$\boxed{est \mu = \bar{x}}$$

As for a point estimate, population mean ( $\mu$ ) can be estimated by means of a sample mean ( $\bar{x}$ ). A sample mean can be calculated using a function AVERAGE in MS Excel.

Elaborated by: Ing. Martina Majorová, Dept. of Statistics and Operations Research, FEM SUA in Nitra

Reference: JAISINGH, L.: Statistics for the Utterly Confused

## Topic 4 Point and interval estimate

---

### Point estimate of variance – bodový odhad rozptylu

$$\boxed{est \sigma^2 = s_1^2}$$

As for a point estimate, population variance ( $\sigma^2$ ) can be estimated by means of a sample variance ( $s_1^2$ ). A sample variance can be calculated using a function VAR in MS Excel.

---

### Point estimate of standard deviation – bodový odhad smerodajnej (štandardnej) odchýlky

$$\boxed{est \sigma = s_1}$$

As for a point estimate, population standard deviation ( $\sigma$ ) can be estimated by means of a sample standard deviation ( $s_1$ ). A sample standard deviation can be calculated using a function STDEV in MS Excel.

---

### Interval estimate – intervalový odhad

#### **Definition:**

Interval estimate is an estimate of population parameter that provides an interval believed to contain the value of the parameter. Interval estimate is not as precise as a point estimate but it is more reliable.

---

### Confidence level – hladina spoľahlivosti

#### **Definition:**

The confidence associated with an interval estimate (usually 0.95 or 0.99, it depends on the significance level, i.e. alpha 0.05 or 0.01).

---

### Confidence interval – interval spoľahlivosti

#### **Definition:**

An interval estimate for an unknown population parameter.

The width of the confidence interval is related to the significance level, standard error, and n (number of observations) such that the following are true:

- **the higher the percentage of accuracy (significance) desired, the wider the confidence interval**
- **the larger the standard error, the wider the confidence interval**
- **the larger the n, the smaller the standard error, and so the narrower the confidence interval**

All other things being equal, a smaller confidence interval is always more desirable than a larger one because a smaller interval means the population parameter can be estimated more accurately.

## Topic 4 Point and interval estimate

---

### Interval estimate of mean - intervalový odhad priemeru (strednej hodnoty)

$$P(\bar{x} - \Delta < \mu < \bar{x} + \Delta) = 1 - \alpha$$

where  $\Delta$  is the sampling error.

- If the sample size is greater than 30 ( $n > 30$ ) then the distribution of the random variable (sample statistic) will be approximated with a normal distribution –  $N(0,1)$ . The sampling error will be calculated as follows:

$$\Delta = u_{(1-\alpha/2)} \cdot \frac{s_1}{\sqrt{n}}$$

A critical value ( $u_{(1-\alpha/2)}$ ) will be calculated using a function NORMSINV in MS Excel.

- If the sample size is lower than 30 ( $n < 30$ ) then the distribution of the random variable (sample statistic) will be approximated with a Student t distribution). The sampling error will be calculated as follows:

$$\Delta = t_{(\alpha;n-1)} \cdot \frac{s_1}{\sqrt{n}}$$

A critical value ( $t_{(\alpha;n-1)}$ ) will be calculated using a function TINV in MS Excel.

---

### Interval estimate of variance - intervalový odhad rozptylu

$$P\left(\frac{(n-1) \cdot s_1^2}{\chi^2_{(\alpha/2;n-1)}} < \sigma^2 < \frac{(n-1) \cdot s_1^2}{\chi^2_{(1-\alpha/2;n-1)}}\right) = 1 - \alpha$$

Critical values for both lower and upper limits ( $\chi^2_{(\alpha/2;n-1)}$ ;  $\chi^2_{(1-\alpha/2;n-1)}$ ) can be calculated using a function CHIINV in MS Excel.

---

### Interval estimate of standard deviation - intervalový odhad smerodajnej (štandardnej) odchýlky

$$P\left(\sqrt{\frac{(n-1) \cdot s_1^2}{\chi^2_{(\alpha/2;n-1)}}} < \sigma < \sqrt{\frac{(n-1) \cdot s_1^2}{\chi^2_{(1-\alpha/2;n-1)}}}\right) = 1 - \alpha$$

Critical values for both lower and upper limits ( $\chi^2_{(\alpha/2;n-1)}$ ;  $\chi^2_{(1-\alpha/2;n-1)}$ ) can be calculated using a function CHIINV in MS Excel.

**Note:** It's much faster to calculate the interval estimate for variance and then calculate the square root of both lower and upper limits 😊.

#### Topic 4 Point and interval estimate

---

**Sampling error (standard error, estimation error) – prípustná chyba odhadu**

$$\Delta = u_{(1-\alpha/2)} \cdot \frac{s_1}{\sqrt{n}} \quad \text{or} \quad \Delta = t_{(\alpha;n-1)} \cdot \frac{s_1}{\sqrt{n}}$$

The lower the desired sampling error the larger the sample size must be.

**Note:** Sampling error is influenced by:

- confidence level - we can influence the confidence level
- (sample) standard deviation - we can't influence the variability within the sample (chosen from the collected set of data)
- sample size - we can influence the sample size

---

**Sample size – veľkosť výberového súboru**

$$n = u_{(1-\alpha/2)}^2 \cdot \frac{s_1^2}{\Delta^2}$$

where,

$\Delta$  and  $s_1$  are given in the text of the example.

A critical value ( $u_{(1-\alpha/2)}$ ) will be calculated using a function NORMSINV in MS Excel.