Chapter 1 – Counting Methods

1.1 Sample Spaces, Events, and Outcomes

Learning Objectives

- Determine the sample space for a given event or series of events
- Produce an organized list of outcomes within a sample space

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A **sample space** is a list of all the possible outcomes that may occur. What might happen when you flip a coin? You will either get heads or tails. What will happen when you roll a single die? You will either get a 1, 2, 3, 4, 5, or 6. The sample space for flipping a coin is S = {heads, tails}. The sample space for rolling a die is S = {1,2,3,4,5,6}

On a coin flip, there are two **outcomes**, heads and tails. There are six different outcomes when considering the **event** of rolling a single die.

Example 1

Suppose you roll two dice. Build a 6 by 6 grid to show the different outcomes that might happen when you add the two dice together.

a) What is the sample space for the different sums that you might get?

b) What is the event for this situation?

c) Based on your grid, which outcome occurs most often?

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Solution

a) The sample space is
S = {2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}.

b) The event is the rolling of the two dice.

c) Notice that a total of 7 can occur 6 different ways. A total of 7 is the most likely outcome.

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Example 2

A child orders breakfast at a restaurant. The restaurant has two choices of drinks: milk and orange juice. The restaurant also has three choices of meat: sausage, ham, and bacon. Suppose the child orders one drink and one type of meat.



a) Give the sample space that shows all the different outcomes for what the child might order.

b) How many different outcomes are possible?

Solution

a) For the drinks, use M = Milk and O = Orange Juice. For the meat, use S = Sausage, H = Ham, and B = Bacon. The child might order MS, MH, MB, OS, OH, or OB. The sample space is S = {MS, MH, MB, OS, OH, OB}. This list can also be generated using a simple grid as shown to the right.

	Milk	**Orange Juice**
Sausage	MS	OS
Ham	MH	OH
Bacon	MB	OB

b) There are six possible outcomes. This can be found simply by counting the number of results within the sample space.

Sometimes, situations can get a bit too complex to simply make a list or build a grid. A **tree diagram** is a visual organizer that is very effective in handling situations with larger numbers of outcomes. We will introduce this concept here, but we will revisit tree diagrams in greater detail in section 1.2.

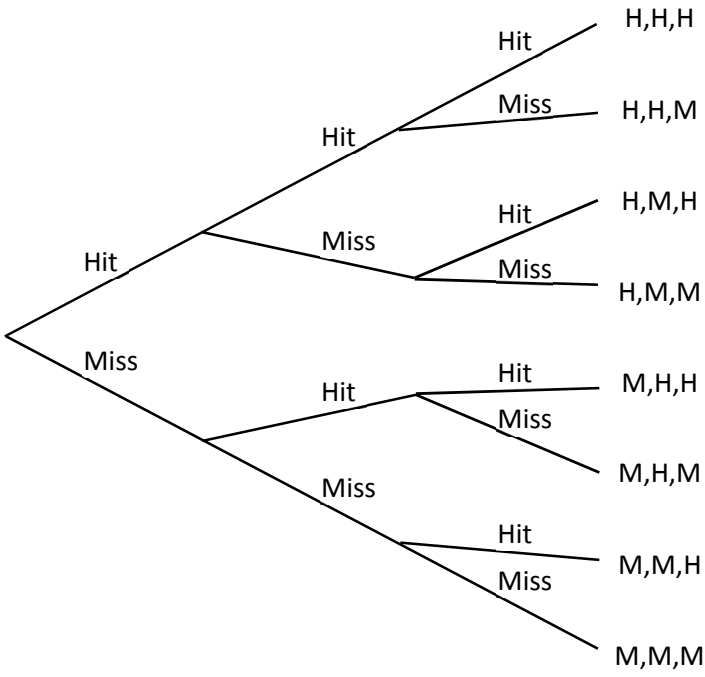
Example 3

A dart player is trying to hit the bulls-eye with each of three darts that he will throw. Each dart will either hit the bulls-eye or miss the bulls-eye. Use a tree diagram to give the sample space for the different outcomes that may occur.

Solution

Build the tree diagram shown to the right to track what might happen.

The sample space is S = {HHH, HHM, HMH, HMM, MHH, MHM, MMH, MMM}



Problem Set 1.1

1) A single coin is flipped two times.

   a) Construct the sample space for this situation.

   b) How many different outcomes are possible?

2) A single coin is flipped three times.

   a) Use a tree diagram to construct the sample space for this situation.

   b) How many different outcomes are possible?

3) A single coin is flipped four times.

   a) Use an organizational strategy to construct the sample space for this situation.

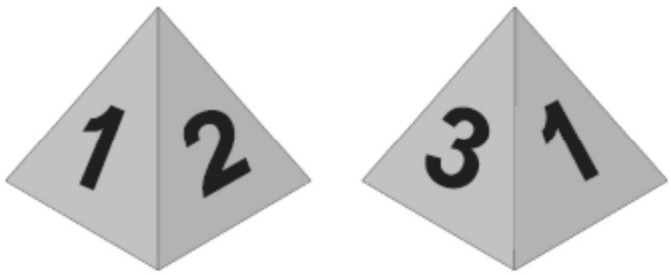
   b) How many different outcomes are possible?

4) Suppose a 4-sided die is rolled one time. What is the sample space for the result of the roll?

5) Suppose two 4-sided dice are rolled and we keep track of the total for the two dice.

   a) Draw a four by four grid that demonstrates the different results for the total of the two dice.

   b) What is the sample space for the possible totals of the two dice?



6) Suppose two 4-sided dice are rolled two times and we keep track of the *product* when the result from the first die is multiplied by the result from the second die.

   a) Draw a four by four grid that demonstrates the different results for the product of the two dice.

   b) What is the sample space for the possible product of the two dice?

   c) How many different outcomes are possible for the product of the two dice?

   d) What outcome occurs most often?

1.2 Fundamental Counting Principle

Learning Objectives

- Apply the Fundamental Counting Principle to determine the number of outcomes
- Create tree diagrams to represent outcomes for a series of events



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The **Fundamental Counting Principle** states that if you wish to find the number of outcomes for a given situation, simply multiply the number of possible outcomes for each step of the event. In Example 2 in section 1.1, the child had two different choices of drink and three different choices of meat. If we multiply 2 times 3, we get 6 which is the total number of outcomes possible. The Fundamental Counting Principle expands to any events with more than just two steps. For example, suppose it turned out that the child also wanted to order eggs and had to choose between the eggs being scrambled or sunny-side up. The Fundamental Counting Principle states that there are $2 \times 3 \times 2$ or 12 ways to order this breakfast.

Fundamental Counting Principle

$$Total\ \#\ of\ Outcomes = (\#\ of\ Choices) \cdot (\#\ of\ Choices) \cdot (\#\ of\ Choices) \cdot \ldots$$

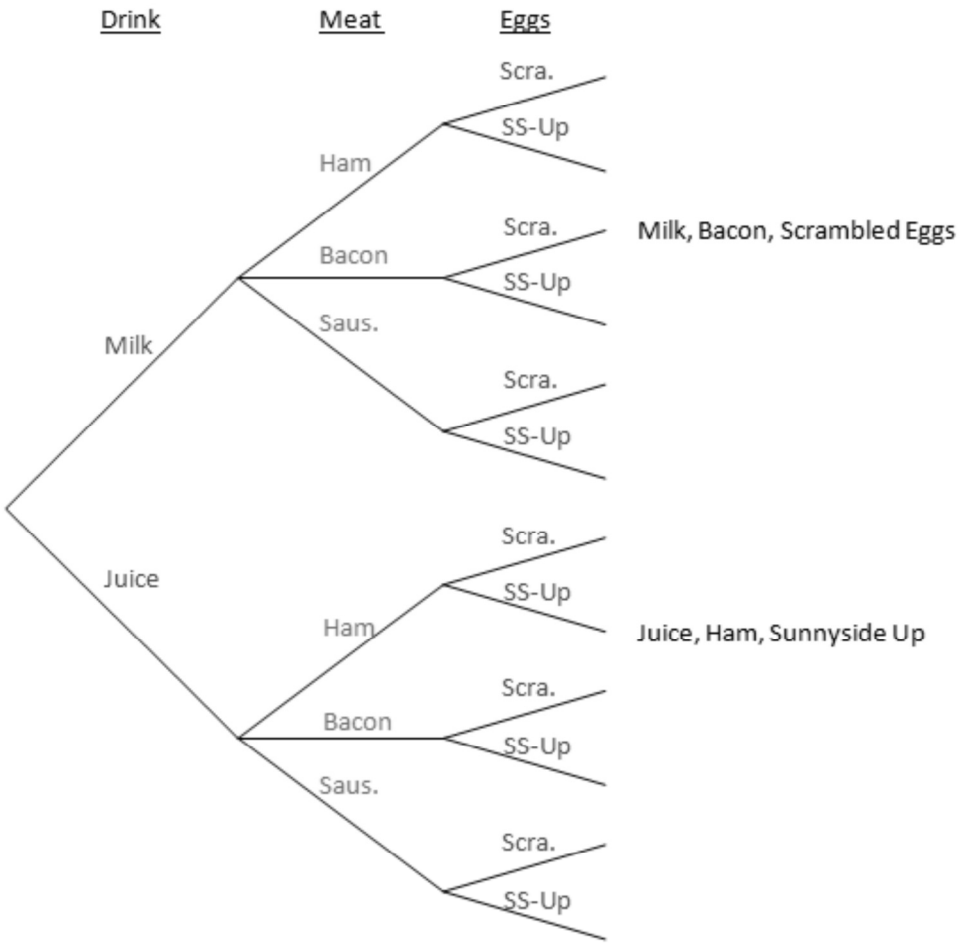
A **tree diagram** will allow us to visually see what is happening in situations like this.

Example 1

Build a tree diagram that shows the different outcomes for what the child might order for breakfast if they have two choices for drinks, three choices for meats, and two choices for eggs.

Solution

The first set of branches of the tree diagram will represent the type of drink, the second set of branches will represent the type of meat, and the third set of branches will represent the type of egg. A diagram of what this will look like is shown on the top of the next page.



We have labeled the ends of two of the branches in the figure above to show what each branch means. For example, one of the labeled branches shows that the child might have ordered milk, bacon, and scrambled eggs.

The Fundamental Counting Principle is critically important especially when considering complex tree diagrams. Our tree diagram above has many branches and it tracks a great deal of material. It ultimately shows us the 12 different possible breakfast orders, but it takes a large amount of organization to successfully complete. Multiplying 2 by 3 by 2 is a much quicker way to find out the total number of possible outcomes.

Example 2

A couple is planning to have 3 children. Consider the different results that might occur in terms of sex.

For example one outcome might be Boy, Boy, Girl (BBG).

a) Using the Fundamental Counting Principle, calculate the number of different outcomes for the children in this family.

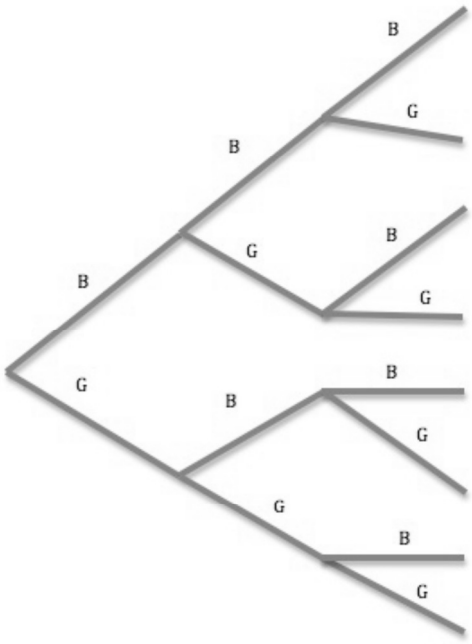
b) Build a tree diagram that shows the different orders of children the couple might have.

c) Build the sample space that shows all the different orders of children the couple might have.

Solution

a) There are 2 choices for the first child, 2 for the second, and 2 for the third. Therefore, there are $2 \times 2 \times 2 = 8$ outcomes for the gender order of the 3 children.

b)



c) In order to be organized, the list will be alphabetized. BBB, BBG, BGB, BGG, GBB, GBG, GGB, GGG. There are a total of 8 outcomes.

There are many other ways to apply the Fundamental Counting Principle. A standard deck of cards has 52 cards as shown to the right. If you are dealt just one card, there are 52 different outcomes.

Standard Deck of 52 Playing Cards

Black cards		***Red cards***	
Clubs	**Spades**	**Hearts**	**Diamonds**
A ♣	A ♠	A ♥	A ♦
2 ♣	2 ♠	2 ♥	2 ♦
3 ♣	3 ♠	3 ♥	3 ♦
4 ♣	4 ♠	4 ♥	4 ♦
5 ♣	5 ♠	5 ♥	5 ♦
6 ♣	6 ♠	6 ♥	6 ♦
7 ♣	7 ♠	7 ♥	7 ♦
8 ♣	8 ♠	8 ♥	8 ♦
9 ♣	9 ♠	9 ♥	9 ♦
10 ♣	10 ♠	10 ♥	10 ♦
Jack ♣	Jack ♠	Jack♥	Jack ♦
Queen ♣	Queen ♠	Queen ♥	Queen ♦
King ♣	King ♠	King ♥	King ♦

Example 3

Suppose you are dealt two cards from a standard deck of 52 cards. How many different outcomes are possible for what your two cards might be?

Solution

We could certainly try drawing a tree diagram but that could get very large quite quickly. The first split alone would have 52 branches on it! On the other hand, if we use the Fundamental Counting Principle, we can simply calculate how many different ways we could be dealt 2 cards from a standard deck. There would be 52 choices for the 1st card and 51 choices for the 2^{nd} card. (Once the first card is dealt, the deck only has 51 cards left in it.) There are $52 \times 51 = 2652$ possible different results for what our two cards might be.

Example 4

How many different 10-digit phone numbers are possible if our only restrictions are that no phone number may begin with a zero or a one and that the 4^{th} digit may not be a zero?

Solution

There are a total of 10 digits available {0, 1, 2, ... 7, 8, 9}. We can't use zero or one for the first digit so there are only 8 choices for the 1st digit. In addition, we can't use a zero for the 4^{th} digit so the 4^{th} digit only has 9 choices. All of the remaining digits will have 10 choices. This gives us $8 \times 10 \times 10 \times 9 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 72 \times 10^8 = 7{,}200{,}000{,}000$ different possibilities for a 10-digit phone number.

Example 5

A teenager is given 5 different jobs that they must do before they may go out to a movie with friends. The jobs are washing the car, starting a load of laundry, vacuuming the family room, taking out the garbage, and putting away the dishes. In how many different orders could the teenager complete these jobs?

Solution

There are five choices the teenager could pick for the first job. Once that job is finished, there are only 4 jobs remaining. Once the 2nd job is completed, there are only 3 choices for the 3rd job. Once the 3rd job is finished, there are only 2 choices for the 4th job and finally there will only be one choice left for the 5th job. There are $5 \times 4 \times 3 \times 2 \times 1 = 120$ different orders that these jobs could be completed. Note that there is a quick way to do this ordered multiplication using **factorials**. $5 \times 4 \times 3 \times 2 \times 1 = 5!$ The 5! is read "Five Factorial". Be sure to locate the factorial key on your calculator.

Factorial Formula

$$n! = n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot (3) \cdot (2) \cdot (1)$$

Problem Set 1.2

Exercises

1) A woman has three skirts, five shirts, and four hats from which to pick an outfit. How many different outfits can she wear if she picks one skirt, one shirt, and one hat?

2) How many different five-digit ZIP codes are possible if the digits can be repeated?

3) How many different five-digit ZIP codes are possible if the digits cannot be repeated?

4) In how many ways can a baseball manager arrange a batting order of nine players?

5) A store manager wishes to display six different brands of laundry soap by lining them up in a row on a shelf. In how many ways can this be done?

6) There are 8 different statistics books, 6 different geometry books, and 3 different trigonometry books being considered for next year. In how many ways can a textbook committee select one of each book?

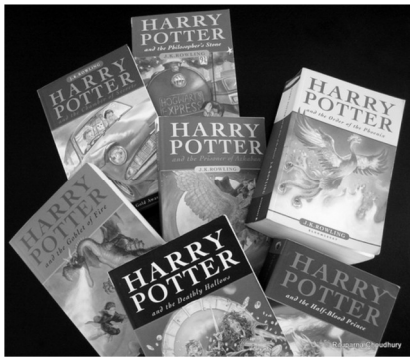
7) At a film festival, there are eight different films that will be shown. In how many different orders can these films be shown?

8) The call letters of a radio station must have four letters. The first letter must be a K or a W. How many different call letter combinations are possible if letters may not be repeated?

9) The call letters of a radio station must have four letters. The first letter must be a K or a W. How many different call letter combinations are possible if letters may be repeated?

10) How many different four-digit ID tags can be made if repeats are allowed?

11) How many different four-digit ID tags can be made if it must start with a 7 and no repeats are allowed?



12) In how many different ways can the Harry Potter series of books (7 books total) be arranged in a row on a shelf?

13) In how many different ways can a manager select a pitcher - catcher combination if the manager has 5 pitchers and 2 catchers from which to choose?

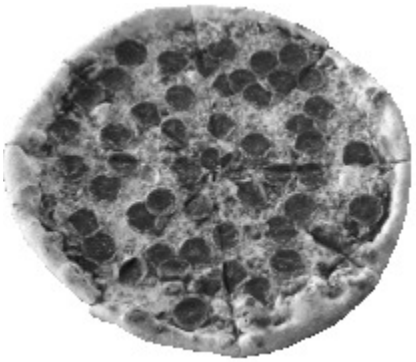
14) A coin is tossed 8 times and the result of each flip is recorded. How many different outcomes are possible for this series of 8 flips?

15) A tile contractor is going to make a tile pattern that contains four tiles. He has six different colors of tile. How many possible different 4-color patterns are possible if no color may be repeated?

16) A tile contractor is going to make a tile pattern that contains four tiles. He has six different colors of tile. How many possible different 4-color patterns are possible if colors may be repeated?

17) Four cards are dealt from a standard deck of 52 cards. In how many different orders of suit could the cards be dealt? For example, one order is Club, Heart, Club, Diamond.

18) A pizza restaurant offers 6 different toppings for their pizzas. How many different pizzas are possible?



19) Use a tree diagram to find all possible outcomes for the result of a series of coin flips if the coin is flipped two times. Write a list of the possible results when complete.

20) The Super-Cool Ice Cream Shoppe sells sundaes, cones, and ice cream bars. The available flavors are either butterscotch or chocolate and you may choose to have it with nuts or without nuts.

    a) Draw a tree diagram to illustrate the different types of ice cream treats that you could order.

    b) How could you find the number of outcomes using the Fundamental Counting Principle?

    c) How many different outcomes are possible?

21) A quiz has four true/false questions on it. Use a tree diagram to show all the different possible answer keys.

22) A box contains a $1 bill, a $5 bill, and a $10 bill. Two bills are selected one after the other without replacing the first bill once it is drawn. Draw a tree diagram to show all the possible ways that these two bills may be picked.

23) The Eagles and Hawks play each other in a hockey tournament. The first team to win two games is the champion. Use a tree diagram to show all different possible outcomes for the tournament.

Review Exercises

24) Consider a situation in which a baseball manager must decide which one of 4 players will pitch (P1, P2, P3, or P4) and which one of 2 players will catch (C1 or C2).

    a) What is the sample space for the possible pitcher/catcher combinations for this situation?

    b) How many different outcomes are possible?

25) A person taking a poll stops two people on the street and asks each of them if they are leaning Democratic, leaning Republican, or undecided for an upcoming election.

    a) Build a grid that shows the different responses that may occur.

    b) How many different outcomes are possible?

1.3 Permutations

Learning Objectives

- Know the definition of a permutation
- Be able to calculate the number of permutations using the permutations formula and with technology
- Understand the connection between the Fundamental Counting Principle and permutations

The Fundamental Counting Principle provides us with a tool that allows us to calculate the number of outcomes possible in many situations. What if the situation is a bit more complex? For many situations, the order that we complete a task does not matter. Ordering milk, bacon, and scrambled eggs in that order is the same as ordering bacon, scrambled eggs, and milk. In that case the order that we make our choices wouldn't matter. However, there are many situations in which the order that we do things does make a difference.

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A **permutation** is a specific order or arrangement of a set of objects or items. What if I wish to call someone on the phone? If I make the call, the order that I punch in the numbers matters so this is an example of a permutation. A good question to ask when deciding if your arrangement is a permutation is **"DOES ORDER MATTER?"** If yes, then you are dealing with a permutation. For example, suppose you order an ice cream sundae and a cherry is put on the bottom, then chocolate sauce on top of that, and finally it is topped off with ice cream. You probably would not be too happy with that particular ice cream sundae. You would likely prefer that the ice cream comes first, followed by the chocolate sauce, and finally the cherry goes on top. Clearly each sundae had the same three ingredients, but they were quite different from one another. Each order that we can make the ice cream sundae is called a permutation.

There is a simple formula for figuring out how many permutations exist when 'r' objects are selected from a set of 'n' objects. The left side of the equation can be read "n P r", just as it looks or "n Permutations of size r".

$$ {}_nP_r = \frac{n!}{(n-r)!} $$

Recall that the exclamation point is a factorial. For example, $5! = 5 \times 4 \times 3 \times 2 \times 1$. Also, be sure to find the permutations command on your calculator.

In our ice cream sundae discussion, 'n' would be 3 because there are 3 items to select from and 'r' would also be 3 because we are going to select all three items. Using the permutations formula, this would be $_3P_3 = \frac{3!}{(3-3)!} = \frac{3!}{0!} = \frac{6}{1} = 6.$ In other words, there are 6 different orders that the ice cream sundae could be made. (Note that 0! is equal to 1.)

Example 1

Suppose you are going to order an ice cream cone with two different flavored scoops. You are going to take a picture of your ice cream cone for use in the school newspaper. The ice cream shop has 5 flavors to choose from; chocolate, vanilla, orange, strawberry, and mint. How many different ice cream cone photos are possible?



Solution

The first question to ask is **"Does Order Matter?**". If it does, then we are dealing with a permutation question. In this case, the order does make a difference. A chocolate on top of vanilla cone looks different than a vanilla on top of chocolate cone. We have five flavors to pick from, so n = 5. We are going to select 2 flavors so r = 2. $_5P_2 = \frac{5!}{(5-2)!} = \frac{5!}{3!} = \frac{120}{6} = 20.$ There 20 different permutations of ice cream cones we could order. The notation representing this situation, $_5P_2$, can be read as "Five 'P' Two" or "Five permutations of size Two". Be sure to perform this calculation on your calculator as well.

In the example above, you could have also found your answer using the Fundamental Counting Principle. There were 5 choices for the 1st flavor and then only 4 choices for the 2nd flavor. There are $5 \times 4 = 20$ ice cream cones possible.

Example 2

Give the value of ${}_6P_3$ by using the formula for permutations. Verify your solution on your calculator.

Solution

$$ {}_6P_3 = \frac{6!}{(6-3)!} = \frac{6!}{3!} = \frac{720}{6} = 120 $$

Example 3

Decide whether each of the situations below involves permutations.

a) A five-card poker hand is dealt from a deck of cards.

b) A cashier must give 3 pennies, 2 dimes, a 5 dollar bill, and a 10 dollar bill back as change for a purchase.

c) A student is going to open a padlock that has a three number combination.

d) A child has red, blue, green, yellow, and orange color crayons and will be coloring a rainbow using each color one time.

Solution

a) The order you get your five cards for a poker hand does not matter. If one of your cards was the ace of spades, it didn't matter if it was the first card or the last card dealt.

b) The order that the cashier gives you $15.23 in change does not matter as long as the total is $15.23.

c) The order you put in the three numbers for the combination makes a difference. If the correct combination is 12-27-19, the padlock will not open if you enter 19-12-27 even though the same three numbers are used.

d) The order that the child colors the rainbow does make a difference. The color pattern red, blue, green, orange, yellow will look different than green, blue, red, yellow, orange.

Problem Set 1.3

Exercises

1) Use the formula for Permutations, ${}_nP_r = \frac{n!}{(n-r)!}$ to find the value for each expression. Confirm each result by using your calculator.

   a) ${}_8P_3$

   b) ${}_4P_4$

   c) ${}_5P_3$

   d) ${}_5P_0$

2) How many 4 letter permutations can be formed from the letters in word *rhombus*?

3) A board of directors composed of eight people must select board officers. In how many ways can a president, vice president, and treasurer be selected from this 8-person board?

4) How many different ID cards can be made if there are six digits on a card and no digit may be used more than once?

5) In how many ways can seven different brands of laundry soap be displayed on a shelf in a store?

6) A child has built a model car and will decorate it with two different racing-stripe stickers, one on each side of the car. In how many ways can the child decorate their car if the child has four different stickers from which to choose?

7) An inspector must select three tests to perform in a certain order on a manufactured part. He has a choice of seven tests. In how many different ways can he perform the three tests?

8) In how many different ways can 4 raffle tickets be selected from 50 tickets if each of the 4 ticket holders wins a different prize?



9) A teacher has five groups in a class. Each group was given the same 5 problems to work on for last night's homework. The teacher has decided that each group will present one of the homework problems on the board during class. In how many ways can the teacher assign problems for the groups for presentation on the board?

10) There are five violinists in an orchestra. Three of them will be selected to play in a trio with a different part for each musician. In how many ways can the music be assigned?

11) There are five violinists in an orchestra. Four of them will be selected to play in a quartet with a different part for each musician. In how many ways can the music be assigned?

12) There are five violinists in an orchestra. All five of them will be selected to play in a quintet with a different part for each musician. In how many ways can the music be assigned?

13) There are five violinists in an orchestra. A piece of music is written so that it can be played with 3, 4, or 5 violinists. Each musician selected to play this piece will play a different part. In how many ways can a group of at least three musicians be selected? Use your answers from problems 10), 11) and 12) to help answer the question.

14) Decide whether each situation below involves permutations. Briefly explain your answers.

    a) Sophia picks three color crayons from a box of 12 crayons to make a picture for her cat, Butterscotch.

    b) A five-digit code is needed to open up an electronic lock on a car.

    c) Mica has 83 pairs of shoes and will pack five of those pairs of shoes to bring with him on vacation.

    d) There are seven steps that a student must follow when preparing cookies during their Family and Consumer Sciences course.

Review Exercises

15) Use the Fundamental Counting Principle to determine the number of different ways a person could order a meal if they are to pick one entree from four choices, one side order from three choices, and one drink from four choices.

16) A student wishes to check out three books from the library. She will check out one historical fiction book, one biography, and one book on art history. Build a tree diagram to show how many ways this can be done if there are two historical fiction books, three biographies, and two books on art history that she is considering checking out.

17) How many different outcomes are possible for the total on a roll of two dice if one die has 6 sides and one die has 4 sides?

1.4 Combinations

Learning Objectives

- Know the definition of a combination
- Be able to calculate the number of combinations using the combinations formula and with technology

We just looked at situations in which order matters. What if order does not matter? Suppose you have a younger brother or sister and your family goes out to a restaurant. All of the children who come to the restaurant get a menu that doubles as a children's activity page. The owner of the restaurant has decided that each child will receive two different colored crayons to use for the activities on the menu. The restaurant happens to carry five colors of crayons: orange, yellow, blue, green, and red.

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This is a situation in which the order that the child gets their two color crayons does not matter. If you gave a child a red crayon and then a blue crayon, it would be the same as if you gave the child a blue crayon followed by a red crayon. As with permutations, the first question to ask is **"Does Order Matter?"**. When the order does not matter, you are dealing with a situation that involves **combinations**.

Example 1

Consider the color crayon problem in the previous situation. Make a list showing all of the different color crayon combinations that might occur. Be organized so as not to repeat any combinations.



Solution

To be organized, use the letters O, Y, B, G, and R to represent the five colors (Orange, Yellow, Blue, Green, and Red). Alphabetizing the list to ensure that we don't skip any combinations gives us BG, BO, BR, BY, GO, GR, GY, OR, OY, RY. Notice that while we have BG, we don't have GB as that would be a repeat because it represents the same two colors. In addition, the list also does not include results like BB because that would represent the same color twice. It appears that there are 10 possible combinations.

As with the Fundamental Counting Principle, we now must ask the question "How can we find the solution quickly?" Making a list works nicely, but it could get a bit messy if the restaurant had 24 colors to choose from instead of 5 because our list would get very long. Out of curiosity, you may have tried ${}_5P_2$. However, you will find that this gives us a result of 20 instead of 10. We will modify the permutation formula for situations involving combinations.

Shown below is the formula for finding how many combinations are possible when order ***does not*** matter.

$$ {}_nC_r = \frac{n!}{r!(n-r)!} $$

As with the permutation formula, the 'n' stands for the number of objects available and the 'r' stands for the number of objects that will be selected.

Example 2

Consider the color crayon problem once again. Use the combination formula to find out the number of different color crayon combinations that are possible.

Solution

In our problem, 'n' is equal to 5 and 'r' is equal to 2. Our calculation would be ${}_5C_2 = \frac{5!}{2!(5-2)!} = \frac{5!}{2!\cdot 3!} = \frac{5!}{2\cdot 6} = \frac{120}{12} = 10.$ Be very careful that you find the result for the denominator before you divide! Find the ${}_nC_r$ command on your calculator and verify that ${}_5C_2$ is indeed equal to 10.

Example 3

Suppose that there are 12 employees in an office. The boss needs to select 4 of the employees to go on a business trip to California. In how many ways can she do this?

Solution

We first ask whether the order that the employees are selected matters. In this case, the answer is no because either you will be going on the trip or you won't be going. Being the fourth name on the list of people who get to go is just as good as being the first name on the list. We have 12 people to select from and we will be selecting 4 or ${}_{12}C_4 = \frac{12!}{4!(12-4)!} = 495.$ There are 495 possible combination of groups of 4 that might be selected to go on the trip to California.

Problem Set 1.4

Exercises

1) Use the formula for combinations to find the value of each expression. Use a calculator to verify each answer.

   a) ${}_5C_5$

   b) ${}_6C_4$

   c) ${}_3C_0$

   d) ${}_7C_3$

2) In how many ways can 3 cards be selected from a standard deck of 52 cards?

3) In how many ways can three bracelets be selected from a box of ten bracelets?

4) In how many ways can a student select five questions to answer from an exam containing nine questions?



5) In how many ways can a student select five questions to answer from an exam containing nine questions if the student is required to answer the first and the last question?

6) The general manager of a fast-food restaurant chain must select 6 restaurants from 11 for a promotional program. In how many different possible ways can this selection be done?

7) There are 7 women and 5 men in a department. In how many ways can a committee of 4 people be selected?

8) For a fundraiser, a travel agency has donated 5 free vacations to Mexico as grand prizes in a raffle. Suppose that 220 people purchased raffle tickets. In how many different ways can the vacation winners be selected?

9) A high school choir has 27 female and 19 male members. Two students will be selected from the choir to represent the school in the All-State Choir.

   a) In how many ways can the director select two students if she decides both students will be female?

   b) In how many ways can the director select two students if she decides both students will be male?

   c) In how many ways can the director select two students?

   d) Using your answers from a), b) and c), determine how many ways the choir director can select two students such that one student will be a male and one student will be a female?

Review Exercises

10) In how many ways can the team captain of a kickball team arrange the kicking order for the 7 players on the team?

11) An electronic car door lock has five buttons on it and each button has a different letter - A, B, C, D, and E. Suppose the code to unlock the door is 4 letters long.

   a) How many different codes are possible if a letter may be repeated?

   b) How many different codes are possible if a letter may not be repeated?

12) Give the sample space for the different results that may occur if a coin is flipped twice.

13) Decide whether each situation involves permutations.

   a) A teacher must pick two students from a class of 30 to put their answers on the board for problem #11 from last night's homework.

   b) In order to be allowed outside to play in the rain, a 5-year old must correctly put on socks, shoes, and boots.

   c) It is 6 pm right now and Julia has 5 hours of schoolwork to complete by the time she goes to bed. Julia's parents enforce a strict bedtime of 11 pm. She has a paper that will take two hours to write, a math assignment that will take one hour to complete, and a science experiment that will take two hours to finish.

1.5 Mixed Combinations and Permutations

Learning Objectives

- Determine whether a situation involves permutations or combinations
- Understand the mathematical implications of the words 'and' & 'or'

Having covered the basics of combinations and permutations, you are ready to have a mixture of problems with slight variations. A common variation involves an understanding of some key words used in mathematics. Commonly, the word "**and**" indicates multiplication and the word "**or**" indicates addition. Consider the examples below.



https://bit.ly/probstatsSection1-5
1 video link

Example 1

In how many ways can committee of 3 people be chosen if there are 8 men and 4 women available for selection and we require that two men and one woman be on the committee?

Solution

The order that we place the people on a committee does not matter. It makes no difference if you are the first person or the last person selected for the committee. Either you are on the committee or you are not on the committee, therefore this is a combination question. Notice that we want two men **and** one woman. The word 'and' indicates multiplication. In other words, we will look for the product of how many ways we can select two men from eight and one woman from four. ${}_8C_2 \times {}_4C_1$ = 28 × 4 = 112. There are 112 ways to select this committee of 3 people.

Example 2

In how many ways can a committee of 5 people be chosen if there are 7 men and 5 women available for selection and we require *at least* 4 women on the committee?

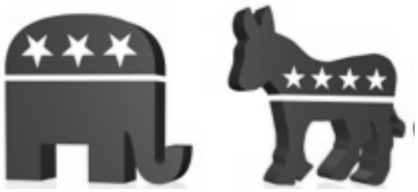
Solution

We first ask "Does order matter?". In this case, the order that someone is placed on a committee does not matter. Either you are on the committee or you are not. Once again, we are dealing with a combination question. The key phrase in this example is *at least*. This can be interpreted to mean that we either select 4 women **and** 1 man **or** 5 women **and** 0 men.

Remember that the word 'and' indicates multiplication and the word 'or' indicates addition. It looks like we are going to have some addition and some multiplication in this problem. ${}_5C_4 \times {}_7C_1 + {}_5C_5 \times {}_7C_0$ = 5 × 7 + 1 × 1 = 35 + 1 = 36. There are 36 ways to put this committee together.

Example 3

In a certain country, there are two political parties that will be represented on a ballot. Each party is responsible for nominating both a presidential and vice-presidential candidate for the ballot. In the first party, there are 6 candidates available and in the second party there are 5 candidates available. How many different ballots are possible?



Solution

The order that we select the candidates does make a difference. Selecting party member 'A' for a presidential candidate and party member 'B' for a vice-presidential candidate is different than selecting party member 'B' for a presidential candidate and party member 'A' for a vice-presidential candidate. Therefore, this is a permutations question. Since we will select candidates from the first party **and** candidates from the second party, we expect there to be multiplication in this problem as well.

${}_6P_2 \times {}_5P_2 = 30 \times 20 = 600$. There are 600 different possible ballots.

Problem Set 1.5

Exercises

1) State how you can tell the difference between a combination problem and a permutation problem.

2) Your closet contains 10 different styles of shoes. In how many ways can you pick out five different styles of shoes for the school week if you don't care which day of the week you wear each style?

3) Your closet contains 10 different styles of shoes. In how many ways can you pick out five different styles of shoes for the school week if you do care which day of the week you wear each style?



4) You are drawing a rainbow using five different colored crayons from your box of 24 colors. In how many ways can you draw a rainbow if the first color you pick will be the top layer and so on?

5) In how many ways can you pick 5 different colors from a box of 24 colors to draw a rainbow?

6) Suppose 5 cards are dealt from a standard deck of 52 cards.
   a) How many 5-card hands are possible?
   b) In how many different orders can 5 cards be dealt from a standard deck?

7) Suppose the majority party in a foreign country must select a prime minister and secretary of state from an eligible group of 36 party members. In how many ways can this be done?

8) There are 7 women and 5 men in a department. Four people are needed for a focus group.
   a) In how many ways can a group of 4 people be selected?
   b) In how many ways can this group be formed if it must include *exactly* 2 men and 2 women?
   c) In how many ways can this group be formed if it must include *at least* 2 women?

9) In how many ways can 3 cars and 4 trucks be selected from 8 cars and 11 trucks for inspection?

10) In a train yard there are 4 tanker cars, 12 boxcars, and 7 flatcars available for a train. In how many ways can the train cars be selected so the train has 2 tanker cars, 5 boxcars, and 3 flatcars?

11) Flakes-R-Us cereal comes in two types, Sugar Sweet and Touch O'Honey. If a researcher has ten boxes of each type, how many ways can she select two boxes of each for a quality control test?

12) In how many ways can a jury of 12 people be selected from a pool of 12 men and 10 women?

13) In how many ways can a jury of 6 men and 6 women be selected from 12 men and 10 women?

14) A corporation president must select a manager and assistant manager for each of two stores. In how many ways can this be done if the first store has 9 employees and the second store has 7 employees?

15) Suppose the required sophomore class schedule consists of 2 math classes, 2 social studies classes, and 1 reading class. In how many ways can a student select classes if there are 4 different math classes, 5 different social studies classes, and 2 different reading classes from which to choose?

16) In how many ways can 6 people be assigned to 3 offices if there will be two people in each office?

Review Exercises

17) Use the formula for combinations to find the value of ${}_7C_3$.

18) In how many ways can the letters in the word 'magic' be arranged?

19) How many different sums are possible when two 4-sided dice are rolled?

20) A teacher will select three students to work problems on the board from her class of 34 students. In how many ways can this be done if the three problems to be worked are #11, #14, and #26?

1.6 Chapter 1 Review



https://bit.ly/probstatsSection1-6
1 video link

There are three primary counting methods that are commonly used in probability: the Fundamental Counting Principle, combinations, and permutations. The Fundamental Counting Principle states that to find the number of outcomes for a given situation, simply multiply the number of ways each step of an event may occur by each other. When deciding whether to use combinations or permutations, you must ask if the order matters. If order matters, use permutations, otherwise use combinations. When working with counting outcomes, it is often helpful to have an organizational strategy. Common strategies involve making organized lists, grids, or tree diagrams. Using an organizational strategy will make it much easier for you to come up with the correct sample space.

Review Exercises

1) Suppose that two 5-sided dice are rolled.
   a) Draw a grid showing all the outcomes for the different totals that may occur.
   b) Use {brackets} to write down the sample space.
   c) Suppose a friend offers to play a game in which you are paid $4 any time a total divisible by 4 occurs. Otherwise you pay your friend $2. If you decide to play, would you expect to win money or lose money? Use your grid from part a) to help explain your answer.

2) The lunch at The Diner has a choice of ham, turkey, or roast beef on rye or white bread with coffee or milk. Draw a tree diagram that illustrates what a person might have for lunch if they pick only one meat, one bread, and one drink.

3) Find the value for each expression below. Show your work by hand and use your calculator to verify your results.
   a) 5!
   b) ${}_6P_3$
   c) ${}_7C_5$
   d) (5 – 2)!
   e) 4! – 2!

4) There are four runners in a race. In how many ways can the runners finish the race?

5) A store has eighteen outfits available for a window display, but only six outfits can fit at one time in the display. In how many different ways can 6 outfits be arranged?

6) Paul has three baseballs and four bats. How many possible ball and bat combinations can he choose if he picks one of each?

7) How many license plates are possible if each plate must have three letters followed by three digits and repeats are allowed?



8) How many license plates are possible if each plate must have three letters followed by three digits and repeats are not allowed?

9) There are twenty candidates in the Mr. Minnesota contest. In how many ways could the judges choose the winner, first-runner up, and second-runner up?

10) The yearbook editor must select two photos out of 42 junior photos and two out of 45 senior photos for a page in the yearbook. In how many ways can the photos be selected?

11) A homeless shelter has decided to purchase all new kitchen appliances. They need one oven, one refrigerator, and one dishwasher. The appliance store has 7 brands of ovens, 6 brands of refrigerators, and 5 brands of dishwashers. In how many brand arrangements can the shelter purchase their appliances?



12) An ice cream shop has 8 different flavors of ice cream available. How many 2-scoop cones can be made if you are allowed to have the same flavor for both scoops?

13) An ice cream shop has 8 different flavors of ice cream available. How many 2-scoop cones can be made if you decide not to have the same flavor for both scoops?

14) Suppose a jury of 12 is being selected from a pool of 20 candidates. In how many ways can the jury be selected?

15) Suppose a jury of 12 is being selected from a pool of 13 men and 7 women. In how many ways can the jury be selected if the judge states that the jury must contain *exactly* 5 women?

16) Suppose a jury of 12 is being selected from a pool of 13 men and 7 women. In how many ways can the jury be selected if the judge states that the jury must contain *at least* 5 women?

17) In how many ways can I put together an outfit if I have 7 shirts, 5 pairs of pants, and 4 hats from which to choose assuming I select one of each type of item?

18) For $7.99, a restaurant will sell you their lunch special. The special is a hamburger or chicken sandwich, onion rings or fries, and soda or coffee.

    a) Make a tree diagram showing the different ways a customer may order the lunch special.

    b) How many outcomes are there? Use the Fundamental Counting Principle to justify your answer.

Image References

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Tetrahedral Dice http://www.bbc.co.uk

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License Plate http://www.15q.net

Appliances http://homeappliancesblog.com/

Appendices

Appendix A – Tables

Appendix A, Part 1 - Random Digit Table

Line 101	19223	95034	05756	28713	96409	12531	42544	82853
Line 102	73676	47150	99400	01927	27754	42648	82425	36290
Line 103	45467	71709	77558	00095	32863	29485	82226	90056
Line 104	52711	38889	93074	60227	40011	85848	48767	52573
Line 105	95592	94007	69971	91481	60779	53791	17297	59335
Line 106	68417	35013	15529	72765	85089	57067	50211	47487
Line 107	82739	57890	20807	47511	81676	55300	94383	14893
Line 108	60940	72024	17868	24943	61790	90656	87964	18883
Line 109	36009	19365	15412	39638	85453	46816	83485	41979
Line 110	38448	48789	18338	24697	39364	42006	76688	08708
Line 111	81486	69487	60513	09297	00412	71238	27649	39950
Line 112	59636	88804	04634	71197	19352	73089	84898	45785
Line 113	62568	70206	40325	03699	71080	22553	11486	11776
Line 114	45149	32992	75730	66280	03819	56202	02938	70915
Line 115	61041	77684	94322	24709	73698	14526	31893	32592
Line 116	14459	26056	31424	80371	65103	62253	50490	61181
Line 117	38167	98532	62183	70632	23417	26185	41448	75532
Line 118	73190	32533	04470	29669	84407	90785	65956	86382
Line 119	95857	07118	87664	92099	58806	66979	98624	84826
Line 120	35476	55972	39421	65850	04266	35435	43742	11937

Line 121	71487	09984	29077	14863	61683	47052	62224	51025
Line 122	13873	81598	95052	90908	73592	75186	87136	95761
Line 123	54580	81507	27102	56027	55892	33063	41842	81868
Line 124	71035	09001	43367	49497	72719	96758	27611	91596
Line 125	96746	12149	37823	71868	18442	35119	62103	39244
Line 126	96927	19931	36089	74192	77567	88741	48409	41903
Line 127	43909	99477	25330	64359	40085	16925	85117	36071
Line 128	15689	14227	06565	14374	13352	49367	81982	87209
Line 129	36759	58984	68288	22913	18638	54303	00795	08727
Line 130	69051	64817	87174	09517	84534	06489	87201	97245
Line 131	05007	16632	81194	14873	04197	85576	45195	96565
Line 132	68732	55259	84292	08796	43165	93739	31685	97150
Line 133	45740	41807	65561	33302	07051	93623	18132	09547
Line 134	27816	78416	18329	21337	35213	37741	04312	68508
Line 135	66925	55658	39100	78458	11206	19876	87151	31260
Line 136	08421	44753	77377	28744	75592	08563	79140	92454
Line 137	53645	66812	61421	47836	12609	15373	98481	14592
Line 138	66831	68908	40772	21558	47781	33586	79177	06928
Line 139	55588	99404	70708	41098	43563	56934	48394	51719
Line 140	12975	13258	13048	45144	72321	81940	00360	02428
Line 141	96767	35964	23822	96012	94591	65194	50842	53372
Line 142	72829	50232	97892	63408	77919	44575	24870	04178
Line 143	88565	42628	17797	49376	61762	16953	88604	12724
Line 144	62964	88145	83083	69453	46109	59505	69680	00900
Line 145	19687	12633	57857	95806	09931	02150	43163	58636
Line 146	37609	59057	66967	83401	60705	02384	90597	93600
Line 147	54973	86278	88737	74351	47500	84552	19909	67181
Line 148	00694	05977	19664	65441	20903	62371	22725	53340
Line 149	71546	05233	53946	68743	72460	27601	45403	88692
Line 150	07511	88915	41267	16853	84569	79367	32337	03316

Appendix A, Part 2 – The Normal Distribution Table

For z-scores with z less than or equal to zero

Table 8.1

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3829
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

For z-scores with z greater than or equal to 0

Table 8.2

z	0.0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0..6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

Appendix A, Part 3 – A standard deck of 52 cards

Black cards		*Red cards*	
Clubs	**Spades**	**Hearts**	**Diamonds**
A ♣	A ♠	A ♥	A ♦
2 ♣	2 ♠	2 ♥	2 ♦
3 ♣	3 ♠	3 ♥	3 ♦
4 ♣	4 ♠	4 ♥	4 ♦
5 ♣	5 ♠	5 ♥	5 ♦
6 ♣	6 ♠	6 ♥	6 ♦
7 ♣	7 ♠	7 ♥	7 ♦
8 ♣	8 ♠	8 ♥	8 ♦
9 ♣	9 ♠	9 ♥	9 ♦
10 ♣	10 ♠	10 ♥	10 ♦
Jack ♣	Jack ♠	Jack♥	Jack ♦
Queen ♣	Queen ♠	Queen ♥	Queen ♦
King ♣	King ♠	King ♥	King ♦

Appendix A, Part 4 - Results for the total of two 6-sided dice

+	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Appendix B – Glossary and Index

95% confidence statement – Page 120, Section 4.4

A confidence statement is a summary statement of the findings of a study. All confidence statements have the form 'We are 95% confident that the true proportion of (parameter of interest) will be between (low value of confidence interval) and (high value of confidence interval).'

Back to Back Stem Plots – Page 184, Section 5.6

A stem plot in which two sets of numerical data share the stems in the middle, with one set having its leaves going to the right and the other set having its leaves going to the left.

Bar Graph – Page 137, Section 5.1

A graph in which each bar shows how frequently a given category occurs. The bars can go either horizontally or vertically. Bars should be of consistent width and need to be equally spaced apart. The categories may be placed in any order along the axis.

Bias - Page 95, 103, Section 4.1, 4.2

Bias occurs when a measurement repeatedly reports values that are either too high or too low.

Bin Width

See Class Size

Bivariate Data - Page 200, Section 6.1

Numerical data that measures two variables.

Blind Study - Page 126, Section 4.5

A study in which the subject does not know exactly what treatment they are getting.

Block Design - Page 128, Section 4.5

A study in which subjects are divided into distinct categories with certain characteristics (for example, males and females) before being randomly assigned treatments in an experiment.

Box Plot (Box and Whisker Plot) - Page 171, Section 5.5

A display in which a numerical data set is divided into quarters. The 'box' marks the middle 50% of the data and the 'whiskers' mark the upper 25% and lower 25% of the data.

Categorical Variable - Page 93, 136, Section 4.1, 5.1

Variables that can be put into categories, like favorite color, type of car you own, your sports jersey number, etc...

Census - Page 97, 101, Section 4.1, 4.2

A special type of study in which data is gathered from every single member of the population.

Center - Page 147, 156, Section 5.2, 5.3

Typically, it is the mean, median, or the mode of a data set. In a normal distribution curve the mean, median, and mode all mark the center. If a data set is skewed or has outliers, it is standard practice to use the median as the center.

Chance Behavior - Page 26, Section 2.1

Events whose outcomes are not predictable in the short term, but have long term predictability.

Class Size (Bin Width) - Page 164, Section 5.4

A consistent width that all bars on a histogram have. A quick estimation of a reasonable class size is to roughly divide the range by a value from about 7 to 10.

Coincidence - Page 215, Section 6.2

A relationship between two variables that simply occurs by chance.

Combination - Page 15, Section 1.4

An arrangement of a set of objects in which the order does not matter.

$$_nC_r = \frac{n!}{r!(n-r)!}$$

Common Response - Page 214, Section 6.2

A situation in which two variables have a strong correlation but are actually responding to an additional lurking variable.

Complement of an Event - Page 26, Section 2.1

The probability of an event, 'A', NOT occurring. It can be thought of the opposite of an event and can be notated as Ac or A'. P(A') = 1 – P(A)

Compound Event - Page 33, Section 2.2

An event with two or more steps such as drawing a card and then rolling a die.

Conditional Probability - Page 54, Section 2.5

The probability of a particular outcome happening assuming a certain prerequisite condition has already been met. A clue that a conditional probability is being considered is the word 'given' or the vertical bar symbol, |.

Confidence Interval - Page 119, Section 4.4

The range of answers included within the margin of error. Typically, we use a 95% confidence interval meaning it is very likely (95% chance) that the parameter lies within this range.

Confounding - Page 215, Section 6.2

Occurs when two variables are related, but it is not a clear cause/effect relationship because there may be other variables that are influencing the observed effect.

Context - Page 156, 204 Section 5.3

The specific realities of the situation we are considering. We often consider the labels and units when defining the context.

Contingency Table

See Two-Way Table

Control - Page 125, Section 4.5, 6.1

A researcher in an experiment establishes control when one of the treatment groups receives either a placebo or the currently accepted treatment.

Control Group - Page 125, Section 4.5

A group in an experiment that does not receive the actual treatment, but rather receives a placebo or a known treatment.

Convenience Sample - Page 106, Section 4.2

A biased sampling method in which data is only gathered from those individuals who are easy to access or are conveniently located.

Correlation (r) - Page 210-213, Section 6.2

A statistic that is used to measure the strength and direction of a linear correlation whose values range from -1 to 1. The sign of the correlation (+/-) matches the sign of the slope of the regression equation. A correlation value of 0 indicates no linear relationship whatsoever.

Data - Page 93, Section 4.1

A collection of facts, measurements, or observations about a set of individuals.

Density Curve - Page 236, Section 7.1

A curve that gives a rough description of a distribution. The curve is smooth and always has an area equal to 1 or 100%.

Direct Cause and Effect - Page 214, Section 6.2

A situation in which one variable causes a specific effect to occur with no lurking variables.

Direction - Page 210, Section 6.2

One of three general results reported for a linear regression. It will be reported as either being positive, negative, or 0.

Disjoint

See Mutually Exclusive Events

Dot Plot - Page 154, Section 5.3

A simple display that places a dot above each marked value on the x-axis. There is a dot for each result, so results that occur more than once will be shown by stacked dots.

Double Blind - Page 126, Section 4.5

A study in which neither the person administering the treatments nor the subject knows which treatment is being given.

Empirical Rule (68-95-99.7 Rule) - Page 238, Section 7.1

A rule stating that in a normal distribution, 68% of the data is located within one standard deviation of the mean, 95% of the data is located within two standard deviations of the mean, and 99.7% of the data is located within three standard deviations of the mean.

Event - Page 1, Section 1.1

Any action from which a result will be recorded or measured.

Expected Value - Page 67, Section 3.1

The average result over the long run for an event if repeated a large number of times.

Experiment - Page 97, 124, Section 4.1, 4.5

A study in which the researchers impose a treatment on the subjects.

Explanatory Variable - Page 125, 200, Section 4.5, 6.1

The x-axis variable. It can often be viewed as the 'cause' variable or the independent variable.

Factorial - Page 7, Section 1.2

A number followed by an exclamation point indicated repeated multiplication down to 1. For example, $4! = 4 \times 3 \times 2 \times 1$.

Fair Game - Page 76, Section 3.2

A game in which neither the player nor the house has an advantage. An average player over the long run will neither gain nor lose money. In other words, the expected value of the game is the same as the cost to play the game.

Five-Number Summary - Page 171, Section 5.5

A description of data that includes the minimum, first quartile, median, third quartile, and maximum numbers which can be used to create a box plot.

Form - Page 204, Section 6.1

A general description of the pattern in a scatterplot. Typical descriptions include linear, curved, or random (no specific form).

Frequency Table - Page 137, Section 5.1

A table that shows the number of occurrences in each category.

Fundamental Counting Principle - Page 4, Section 1.2

A rule that states that in order to find the number of outcomes for a multi-step event, simply multiply the number of possibilities from each step of the event.

Histogram - Page 164, Section 5.4

A special bar graph for a numerical data set. In a histogram, each bar has the same bin width and there is no space between consecutive bars. Each bar tracks the number or frequency of results in its given range.

Independent Events - Page 33, Section 2.2

Two events are independent if the outcome of one event does not change the probability for the outcome for the other event.

Individual - Page 93, Section 4.1

This is the person, animal, or object being studied.

Interquartile Range (IQR) - Page 174, Section 5.5

The distance between the lower and upper quartiles. $IQR = Q_3 - Q_1$

Instrument of Measurement - Page 94, Section 4.1

This is the tool used to make measurements. Some examples of instruments include rulers, scales, thermometers, or speedometers.

Intersection of Events - Page 42, Section 2.3

In a Venn Diagram, it includes the results that are members of more than one group simultaneously. We use the symbol, ∩, to indicate the intersection and think of the intersection as those parts of the diagram that include both A and B.

Law of Large Numbers - Page 26, 84, Section 2.1, 3.3

A rule that states that we will eventually get closer to the theoretical probability as we greatly increase the number of times an event is repeated.

Line Graph

See Time Plot

Lurking Variable - Page 124, 214, Section 4.5, 6.2

An additional variable that was not taken into account in a particular situation.

Margin of Error - Page 119, Section 4.4

It is the distance we move above and below the mean to help establish a 95% confidence interval in which we believe the true parameter is located. An approximation for the margin of error for a 95% confidence interval is M.O.E $= \pm \frac{1}{\sqrt{n}}$ where n represents the sample size.

Mean (Average) - Page 147, 237, Section 5.2, 7.1

The sum of all the numbers divided by the number of values in a data set. It is also located at the center of a normal distribution and is a good measure of center for symmetric data sets.

Median - Page 147, Section 5.2

The data result in the middle of a data list that has been organized from smallest to largest. If there are two middle data values, then the median is located halfway between those two values. Visually, it marks the spot where half of the area of a graph is below the median and half of the area is above the median. It is common to use the median as your measure of center for skewed data sets or data sets that contain outliers.

Mode - Page 147, Section 5.2

The result that appears most frequently in a data set. It also occurs at the highest point of a density curve.

Multistage Random Sample - Page 104, Section 4.2

A sampling technique that uses randomly selected sub-groups of a population before random selection of individuals occurs.

Mutually Exclusive Events (Disjoint) - Page 41, Section 2.3

Outcomes that cannot occur at the same time. For example, if a single card is drawn from a standard deck, the outcomes of a diamond and a black card are mutually exclusive.

Negative Linear Association - Page 205, Section 6.1

A situation such that as one numerical variable increases, another numerical variable decreases.

Non-Response - Page 108, Section 4.2

A non-sampling error in which individuals selected for a study do not participate or do not answer questions in a survey.

Normal Distribution Curve - Page 237, Section 7.1

A bell-shaped curve that describes a symmetrical data set such that the most frequent results occur near the mean and results become less frequent as you move further from the mean.

Numerical Variable - Page 93, Section 4.1

A variable that can be assigned a numerical value, such as height, distance, or temperature.

Observational Study - Page 97, 124, Section 4.1, 4.5

A study in which researchers do not impose a treatment on the individuals being studied. Data is collected by observing the individuals, surveying the individuals, or collecting data from the individuals from information that is already available. (Observe but do not disturb)

Outcome - Page 1, Section 1.1

A possible result of an event.

Outlier - Page 155, 178, 204, Section 5.3, 5.5, 6.1

A value that is unusual when compared to the rest of a data set. High outliers will be greater than $Q_3 + 1.5$ IQR. Low outliers will be below $Q_1 - 1.5$ IQR.

Parallel Box Plots - Page 183, Section 5.6

Multiple box plots graphed on the same axes to compare multiple data sets.

Parameter - Page 111, Section 4.2

A value that describes the truth about a population. The value is frequently unknown so a parameter is often given as a description of truth.

Permutation - Page 10, Section 1.3

A specific order or arrangement of a set of objects or items. In a permutation, the order in which the items are selected matters.

Pictograph - Page 141, Section 5.1

A bar graph that uses pictures instead of bars. These graphs can be misleading because pictures measure height and width, where bar graphs measure only height. To be effective, all the pictures used must be the same size.

Pie chart - Page 139, Section 5.1

A graph which shows each category as a part of the whole in a circle graph. Pie charts can be used if exactly 100% of the results from a particular situation are known.

Placebo - Page 126, Section 4.5

A fake treatment that is similar in appearance to the real treatment.

Placebo Effect - Page 126, Section 4.5

The placebo effect occurs when a subject starts to experience changes simply because they believe they are receiving a treatment.

Population - Page 101, Section 4.2

The entire group of individuals we are interested in. A population is often described using the word 'all'.

Positive Linear Association - Page 205, Section 6.1

A situation in which as one numerical variable increases, the other numerical variable also increases.

Prime Number - Page 42, Section 2.3

A number that has exactly 2 factors. Remember, 1 is not a prime number!

Probability - Page 26, Section 2.1

The likelihood of a particular outcome occurring.

Probability Model - Page 49, Section 2.4

A table that lists all the values for the outcomes of an event and their respective probabilities. The sum of all the probabilities in a probability model must equal 1.

Processing Errors - Page 109, Section 4.2

An error commonly made due to issues like poor calculations or inaccurate recording of results.

Prospective Studies - Page 124, Section 4.5

A study which follows up with study subjects in the future in an effort to see if there were any long-term effects.

Quartile 1 - Page 172, Section 5.5

The median of all the values to the left of the median. Do not include the median itself in this calculation if the median is one of the data points.

Quartile 3 - Page 172, Section 5.5

The median of all the values to the right of the median. Do not include the median itself in this calculation if the median is one of the data points.

Random Digit Table - Pages 82, 114, Section 3.3, 4.3, Appendix A

A long list of randomly chosen digits from 0 to 9, usually generated by computer software or calculators. A table of random digits can be found in Appendix A, Part 1.

Random Event - Page 26, Section 2.1

An event is random if it does not have short-term predictability but it has long-term predictability. For example, a coin flip is a random event because we do not know what will happen on the next flip, but we can be reasonably sure that about 50% of a long series of flips will land on heads.

Random Sampling Error - Page 107, Section 4.2

Even though a sample is randomly selected, it is entirely possible that a particular result within the population will be over-represented causing us to be significantly different from the parameter. Larger sample sizes reduce random sampling error. The margin of error is stated with most studies to account for random sampling error.

Range - Page 148, 174, Section 5.2, 5.5

A basic description of how spread out a data set is. It is calculated by subtracting the smallest number from the largest number in a data set.

Reliability - Page 95, Section 4.1

How consistently a particular measurement technique gives the same, or nearly the same measurement.

Response Bias - Page 109, Section 4.2

Occurs when an individual responds to a survey with an incorrect or untruthful answer. This type of bias can frequently happen when questions are potentially sensitive or embarrassing.

Response Variable - Page 125, 200, Section 4.5, 6.1

This is the y-axis variable. It can often be thought of as the 'effect' variable or dependent variable.

Retrospective Study - Page 124, Section 4.5

A study in which information about a subject's past is used in the study.

Sample - Page 102, Section 4.2

A representative subset of a population.

Sample Space - Page 1, Section 1.1

A list of all the possible outcomes that may occur.

Sample Survey - Page 97, Section 4.1

A survey that uses a subset of the population in order to try to make predictions about the entire population.

Sampling Frame - Page 103, Section 4.2

A list of all members of a population.

Scatterplot - Page 200, Section 6.1

Graphs that represent a relationship between two numerical variables where each data point is shown as a coordinate point on a scaled grid.

SCOFD - Page 203-206, Section 6.1

This is an acronym used for the description of a scatterplot and stands for Strength, Context, Outliers, Form, and Direction.

Simple Random Sample (SRS) - Page 103, Section 4.2

A sample where all possible groups of a particular size are equally possible. It can be thought of as putting names of all members of a population in a hat and randomly drawing until the desired sample size is reached.

Simulation - Page 82, Section 3.3

A model of a real situation that can be used to make predictions about what might really happen. Often, tables of random digits are used to carry out simulations.

Skewed Distribution - Page 155, 236, Section 5.3, 7.1

A distribution in which the majority of the data is concentrated on one end of the distribution. Visually, there is a 'tail' on the side with less data and this is the direction of the skew.

SOCCS - Page 154-156, Section 5.3

An acronym used to remember the key information to discuss for a distribution: Shape, Outliers, Center, Context, and Spread.

Spread - Page 156, Section 5.3

A way to measure variability of a data set. Common measures of spread are the range, standard deviation, and IQR.

Standard Deviation - Page 174, 237, Section 5.5, 7.1

A measure of spread relative to the mean of a data set. Use this measurement for any data set which is approximately normally distributed.

Statistic - Page 111, Section 4.2

A number that describes results from sample. This number is often a percentage and is used to make an approximation of the parameter.

Stem Plot - Page 157, Section 5.3

A method of organizing data that sorts the data in a visual fashion. The stem is made up of all the leading digits of a piece of data and the leaf is the final digit. No commas or decimal points should be used in a stem plot.

Stratified Random Sample - Page 104, Section 4.2

A sample in which the population is divided into distinct groups called strata before a random sample is chosen from each strata.

Strength - Page 203, 210, Section 6.1, 6.2

One of three measurements reported for a best-fit line that describes how close the data is to being perfectly linear.

Subjects - Page 125, Section 4.5

The individuals that are being studied in an experiment.

Symmetrical Distribution - Page 155, Section 5.3

A distribution in which the left side of the distribution looks like a mirror image of the right side of the distribution.

Systematic Random Sample - Page 104, Section 4.2

A sampling method in which the first selection is made randomly and then a 'system' is used to make the remaining selections. For example, randomly select one person from a list and then select every 14th person after that.

Theoretical Model - Page 26, 82 Section 2.1, 3.3

A model that gives a picture of exactly the frequencies of what should happen in a situation involving probability.

Theoretical Probability - Page 26, Section 2.1

A mathematical calculation of the likelihood that a given outcome will occur.

Time Plot (Line Graph) - Page 145, Section 5.2

A graph that shows how a numerical variable changes over time.

Tree Diagram - Page 2, 4, 48 Section 1.1, 1.2, 2.4

A visual representation of a multi-step event where each successive step branches off from the previous step.

Two-Way Table (Contingency Table) - Page 55, Section 2.5

A table which tracks two characteristics from a set of individuals. For example, we might track gender and grade of all the students in your high school.

Undercoverage - Page 107, Section 4.2

A sampling error in which an entire group or groups of subjects are left out or underrepresented in a study.

Union of Events - Page 41, Section 2.3

A union includes all results that are in either one category, another category, or both categories in a Venn diagram. We use the symbol $\cup$ and can think of a union as anything belonging to either A, B, or both A and B.

Validity - Page 95, Section 4.1

A measurement technique is valid if it is a reasonable way to collect data.

Variables - Page 93, Section 4.1

Characteristics about the individuals in a study in which researchers might have interest.

Venn Diagrams - Page 29, 42, Section 2.1, 2.3

Diagrams that represent outcomes or categories using intersecting circles.

Voluntary Response Survey - Page 105, Section 4.2

A biased sampling method in which participants get to choose whether or not to participate in the survey. The bias occurs because those who are most passionate about an issue will be more likely to respond.

Wording of a Question - Page 108, Section 4.2

The wording of a question can be used to manipulate individuals in a survey such that they are more likely to respond a certain way in the survey which causes bias.

Z-Score - Page 245, Section 7.2

A measure of the number of standard deviations a particular data point is away from the mean in a normal distribution. If a z-score is positive, the value is larger than the mean and if it is negative, it is less than the mean.

Appendix C – Calculator Help

This appendix is not meant to be a full guide for calculators common to students who take this course. Rather, it is intended to highlight some of the locations to access a variety of commands commonly used on a TI-30XS Multiview Scientific Calculator and a TI-84 Plus Graphing Calculator. One online source that can be helpful for those of you with graphing calculator issues can be found on the Prentice Hall website at http://www.prenhall.com/divisions/esm/app/calc_v2/.





Topic 1 - Combinations, Permutations, and Factorials

TI-30 XS Multiview

Access located in the **prb** menu. Enter the value for n, select nCr or nPr, and then enter the value for r.

TI-84 Plus

Access located in the **Math**, **PRB** menu. Enter the value for n, select nCr or nPr, and then enter the value for r

Topic 2 – Random Number Generators

TI-30 XS Multiview

Access located in the **prb** menu. Select rand, enter lowest value, enter highest value.

TI-84 Plus

Access located in the **Math**, **PRB** menu. Select RandInt, enter lowest value, enter highest value, enter number of random values desired.

Topic 3 – Means and Standard Deviations

TI-30 XS Multiview

Enter data into L_1 in the **data** menu. Press **2nd data (stat)** and select 1-Var Stats. Arrow down to find the mean, $\bar{x}$, and the standard deviation, s_x.

TI-84 Plus

Enter data in L1 by selecting **STAT** and **EDIT**. Press STAT and CALC and then select 1-Var Stats. Arrow down to find the mean, $\bar{x}$, and the standard deviation, Sx.

Topic 4 – Correlations, Slopes, and Y-Intercepts

TI-30 XS Multiview

Enter data into L_1 and L_2 in the **data** menu. Press **2nd data (stat)** and select 2-Var Stats for L_1 and L_2. Arrow down to find the slope (a), the y-intercept (b) and the correlation coefficient (r).

TI-84 Plus

Enter data in L1 and L2 by selecting **STAT** and **EDIT**. Press **STAT** and **CALC** and then select LinReg(ax+b). Be sure the Xlist and Ylist are L_1 and L_2. If you wish to store you equation into the Y= menu, press VARS, Y-VARS, Function, and Y_1. If the correlation (r) does not show up, go to **2nd CATALOG** and select DiagnosticOn.

Topic 5 – Normal Distributions

TI-30 XS Multiview

This calculator cannot perform normal distribution calculations.

TI-84 Plus

To find the percent of area in a normal curve, select **2nd DISTR** and select normalcdf(. Enter the lower bound, upper bound, mean, and standard deviation. To find a value from a percentile in a normal distribution, select 2nd DISTR and select invNorm(. Enter the %tile, mean, and standard deviation.

Image References

Random Digit Table http://uwsp.edu/math

Normal Distribution Table http://www.regentsprep.org

TI-30XS Multiview Calculator http://education.ti.com

TI-84 Plus Graphing Calculator http://education.ti.com

Appendix D – Selected Answers

Problem Set 1.1

1a) S={HH, HT, TH, TT}
1b) 4
2a) S={HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}
2b) 8
3a) S={HHHH, HHHT, HHTH, HHTT, HTHH, HTHT, HTTH, HTTT, THHH, THHT, THTH, THTT, TTHH, TTHT, TTTH, TTTT}
3b) 16
4) S={1, 2, 3, 4}
5a) Grid
5b) S={2, 3, 4, 5, 6, 7, 8}
6a) Grid
6b) S={1, 2, 3, 4, 6, 8, 9, 12, 16}
6c) 9
6d) 4

Problem Set 1.2

1) 60
2) 100,000
3) 30,240
4) 362,880
5) 720
6) 144
7) 40,320
8) 27,600
9) 35,152
10) 10,000
11) 504
12) 5,040
13) 10
14) 256
15) 360
16) 1,296
17) 256
18) 64
19) HH, HT, TH, TT
20a) Tree Diagram
20b) Answers will vary.
20c) 12
21) Tree Diagram
22) Tree Diagram
23) Tree Diagram
24a) S = { P1C1, P1C2, P2C1, P2C2, P3C1, P3C2, P4C1, P4C2}
24b) 8
25a) Grid
25b) 9

Problem Set 1.3

1a) 336
1b) 24
1c) 60
1d) 1
2) 840
3) 336
4) 151,200
5) 5,040
6) 12
7) 210
8) 5,527,200
9) 120
10) 60
11) 120
12) 120
13) 300
14a) No
14b) Yes
14c) No
14d) Yes
15) 48
16) Tree Diagram, 12
17) 9

Problem Set 1.4

1a) 1
1b) 15
1c) 1
1d) 35
2) 22,100
3) 120
4) 126
5) 35
6) 462
7) 495
8) 4,102,565,544
9a) 351
9b) 171
9c) 1035
9d) 513
10) 5,040
11a) 625
11b) 120
12) S={HH, HT, TH, TT}
13a) No
13b) Yes
13c) No

Problem Set 1.5

1) Answers will vary.
2) 252
3) 30,240
4) 5,100,480
5) 42,504
6a) 2,598,960
6b) 311,875,200
7) 1,260
8a) 495
8b) 210
8c) 420
9) 18,480
10) 166,320
11) 2,025
12) 646,646
13) 194,040
14) 3,024
15) 120
16) 90
17) 35
18) 120
19) 7
20) 35,904

Chapter 1 Review

1a) Grid
1b) S={2, 3, 4, 5, 6, 7, 8, 9, 10}
1c) Lose
2) Tree Diagram

3a) 120
3b) 120
3c) 21
3d) 6
3e) 22
4) 24
5) 18,564 or 13,366,080
6) 12
7) 17,576,000
8) 11,232,000
9) 6,840
10) 852,390
11) 210
12) 64
13) 56
14) 125,970
15) 36,036
16) 49,335
17) 140
18a) Tree Diagram
18b) 8